PHYS-434 – Physics of photonic semiconductor devices

Series 1 – RHEED and the square quantum well

1. The figure below shows RHEED oscillations for the growth of GaAs on the (001) plane by MBE.

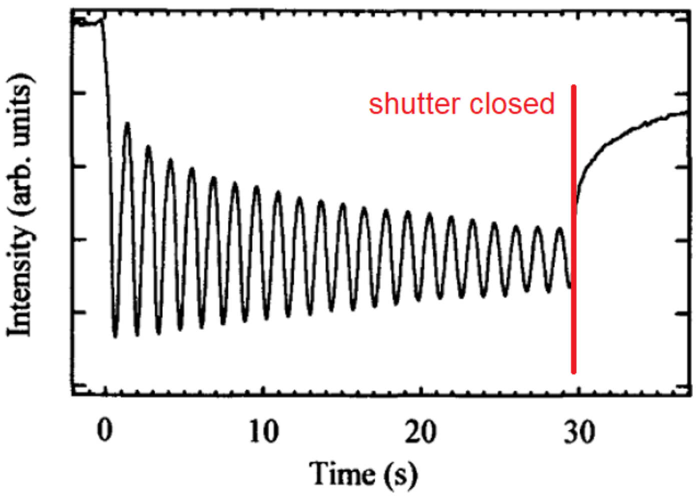

Figure 1: RHEED spot intensity evolution with time for the growth of GaAs on (001).

   (a) Explain why, for monolayer-by-monolayer growth, oscillations in the intensity of RHEED spots are seen.
   (b) Given that GaAs has a lattice parameter $a = 5.653$ Å, calculate the growth rate given by the above RHEED signal in $\mu m\,h^{-1}$.
   (c) Suggest why, in the figure, the amplitude of oscillations decreases with time.
   (d) Sketch how RHEED intensity would be expected to evolve over time for Stranski-Krastanov growth.
   (e) Why can RHEED not be used to monitor MOCVD growth?

2. Let us consider an electron (effective mass m_e^*) in the conduction band of a semiconductor quantum well of width L. Due to the geometry, we can separate the total electron wavefunction Ψ into a wavefunction which describes the behaviour of the electron within the plane of the quantum well, ϕ, and one which describes the behaviour across the quantum well, χ_n:

$$\Psi = \phi(x, y)\chi_n(z) \tag{1}$$

where ϕ has the usual unquantised, nearly-free electron form; as such we focus on χ_n, which is directly influenced by the quantum well. If we assume the potential barrier on either side of the well is infinite, while the potential inside the well is constant and equal to zero, this situation reduces to the well-known one dimensional "particle in a box" scenario.

   (a) Write down the form of the time-independent Schrödinger equation as it applies to the electron inside the quantum well ($0 \leq z \leq L$), along with the boundary conditions χ_n must satisfy.

(b) Show that the general solution $\chi_n = A\sin(k_n z) + B\cos(k_n z)$ satisfies the Schrödinger equation from part (a), and hence show that the energy of the electron within the well is given by:

$$\varepsilon_n = \frac{\hbar^2 k_n^2}{2m_e^*} \tag{2}$$

(c) Next, by applying the boundary conditions from part (a), show that the wavevector of the electron, k_n, must be quantised and hence show the full normalised forms of χ_n are given by:

$$\chi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi z}{L}\right) \tag{3}$$

where n is any positive integer. Hence express their energies ε_n in terms of $n, \pi, \hbar, m_e^*$ and L.

In reality, the well potential barrier will not be infinite, but rather have a finite value V_0. In this case, continuity boundary conditions at the edges of the quantum well yield (for even and odd wavefunctions respectively):

$$\frac{k}{m_A^*}\tan\left(\frac{kL}{2}\right) = \frac{\kappa}{m_B^*} \qquad \frac{k}{m_A^*}\cot\left(\frac{kL}{2}\right) = \frac{-\kappa}{m_B^*} \tag{4}$$

where:

$$k = \frac{\sqrt{2m_A^*\varepsilon_n}}{\hbar} \qquad \kappa = \frac{\sqrt{2m_B^*(V_0 - \varepsilon_n)}}{\hbar} \tag{5}$$

k and m_A^* are the wavevector and effective mass of the electron inside the well, while κ and m_B^* are the wavevector/effective mass in the barriers. Equations (4) are clearly only true for certain values of k and κ; thus these equations show the system is still quantised.

(d) **Trickier question.** Assuming for simplicity that $m_A^* = m_B^* = m_e^*$, show by graphical means that the number of bound states contained in the well, N, is given by:

$$N = \left\lceil \left(\frac{2m_e^* V_0 L^2}{\pi^2\hbar^2}\right)^{\frac{1}{2}} \right\rceil \tag{6}$$

Comment on this expression.

Hint: use the substitution $u = \kappa L/2$ and $\nu = kL/2$ to make the graphical interpretation more readily apparent. $\lceil$ and $\rceil$ are "round up to the nearest integer" brackets, i.e., a ceiling function.

(e) Considering an $Al_{0.3}Ga_{0.7}As$/GaAs/$Al_{0.3}Ga_{0.7}As$ quantum well of width $L = 12$ nm, calculate the number of bound states for an electron in the conduction band at 0 K.

[$E_g^{GaAs} = 1.52$ eV, $E_g^{AlAs} = 3.13$ eV, $m_e^* = 0.067m_0$ for GaAs where m_0 is the rest mass of an electron. To find the bandgap of $Al_{0.3}Ga_{0.7}As$, use the empirical equation: $E_g^{Al_xGa_{1-x}As} = xE_g^{AlAs} + (1-x)E_g^{GaAs} - x(1-x)C$ where C is the "bowing parameter" and is taken equal to $(-0.127 + 1.31x)$]